rfunction_com2[12,41]

The '$\{f \mid x{:}A \rightarrow B\}$' type is the type of functions with domain 'A' and range type '$B(f,x)$' on argument 'x', where 'f' is the function itself. That is, this is a type of highly dependent functions, whose range type depends on calls to the function itself. In this version, the domain type 'A' must be well-founded with some relation 'R', and the calls to the function in the range 'B' must always obey 'R'.

Here is a formal description of the type, along with its members.

We define '$\{f \mid x{:}A \rightarrow B\}$' to be a type
with membership '*phi*' iff
  there is an '*alpha*' such that
  1. 'A' is a type with membership '*alpha*'
  2. 'A' is well-founded with respect to some partial order 'R'
  3. for any 'a' such that '$alpha(a)$'
     there is a '*gamma_?*' such that
       for all '$b\ R\ a$',
       1. '$\{f \mid x{:}\{c{:}A \mid c\ R\ b\} \rightarrow B\}$' is a type with membership '*gamma_b*',
       2. there is a '*beta_?*' such that
          1. for any 'g' such that '$gamma\_b(g)$',
             '$B(g,b)$' is a type with membership '*beta_g*'
          2. for any 'f', '$phi(f)$' iff
             for all 'a' such that '$alpha(a)$', '$beta\_a(f(a))$'